IB Mathematics Analysis and Approaches SL

Internal Assessment: Mathematics Exploration

To Explore if the Density of a Violin Affects its Price

Session: May 2024

Pages: 17

INTRODUCTION

Before my grandfather passed away in 2012, he always enjoyed listening to classical music. However, I was too young and physically incapable of playing a musical instrument for him. When I finally turned eight, I dedicated myself to practicing the violin every day and continuing my family's generational love for music. Although my violin is enjoyable to listen to when played virtuosically, it has many materialistic imperfections that a more expensive violin does not have. For instance, Stradivarius violins from Italy are the most expensive instruments valued at millions of $USD (Walton, 2022). Makers of stringed instruments or "luthiers" handcrafted the wood design and thickness of these violins for quality sound (Stradivarius). Therefore, I wondered what would be more costly: a violin with a lower or higher density? To investigate the question, I will compare the differences in wood density between a budget violin and an expensive violin to analyze if the density affects its price. Thus, I aim **to explore if the density of a violin affects its price**. To begin, I will use functions I have learned in class and apply transformation rules to model the violins' shapes. Next, I took a picture of the most expensive Stradivarius violin in the world, called the "Messiah", which stands at 20 million $USD (Walton, 2022). The violin exemplifies a drastic price gap compared to a budget violin. The price gap of the "Messiah" compared to the budget violin will allow me to explore if there are any differences in density. Then, I will take a picture of my personal violin to illustrate the budget violin. Both violins are full size or "$\frac{4}{4}$" relative to a violin's standardized scale. Therefore, it shows more reliable and consistent results. Lastly, I will manually take the chinrest and shoulder rest off of my violin so that the graphs can be measured without blocking the surface view of the instrument; to begin transforming the functions.

MODELLING THE MODERN VIOLIN

I created my model using GeoGebra to represent the points in centimeters (cm) while following the violin's curvature and the graphs' transformations. To draw the proper functions: $y = a[b(x - d)]^2 + c$, $y = a\sqrt{b(x - d)} + c$, and $y = a\sin[b(x - d)] + c$, I can use transformation rules accordingly. I measured the length of the violin to be 35cm. Therefore, I transformed the graphs between the x values of 0 and 35.

f(x) = $-0.07(x - 7.65)^2 + 10.3$

g(x) = $\sqrt{28x}$

h(x) = $0.3(x - 13.2)^2 + 8.65$

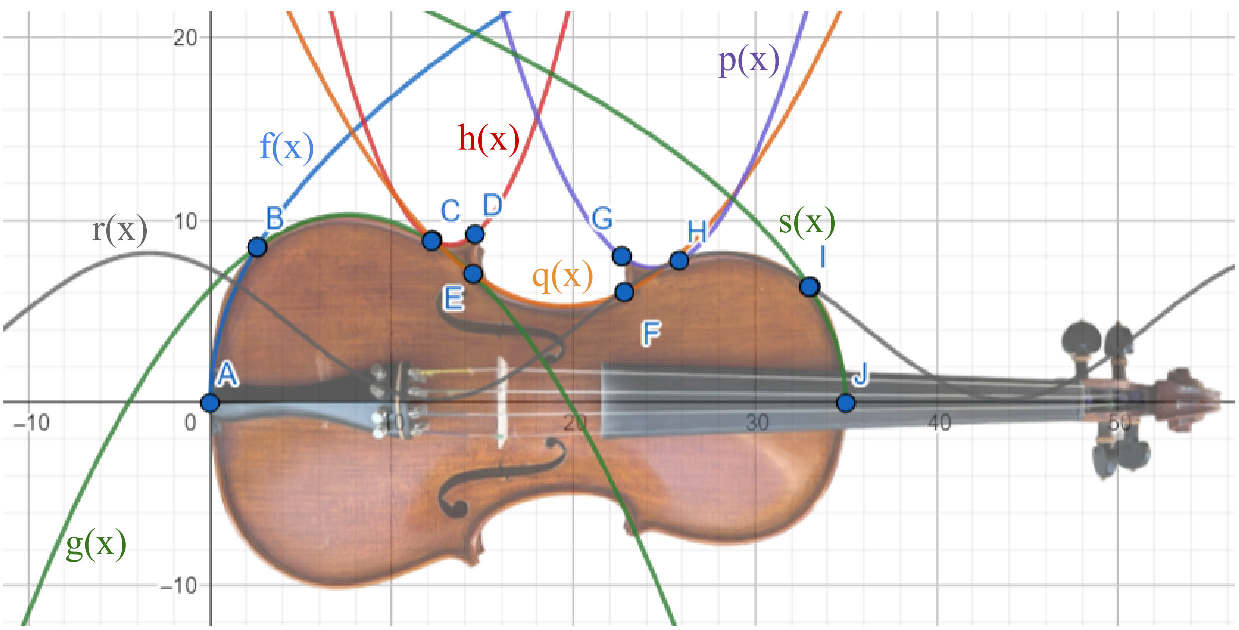
q(x) = $0.07(x - 19.5)^2 + 5.3$

p(x) = $0.2(x - 24.45)^2 + 7.4$

r(x) = $4\sin[0.2(x + 11.2)] + 4.2$

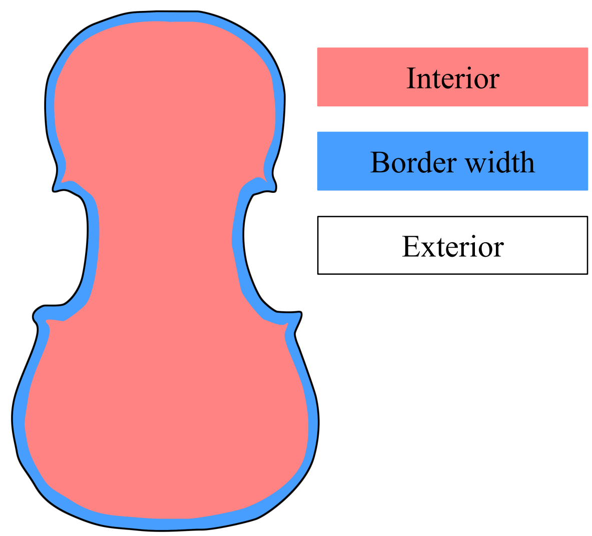
s(x) = $\sqrt{-20(x - 35)}$

Modelling the area of my modern violin (GeoGebra)



With this approach, I can find half the area of my violin underneath each graph above the x-axis. By identifying two points along each graph, I will then apply integration rules based on the type of function. To find the total area, I can calculate the sum of all the areas between each set of two consecutive points and then multiply the value by 2 for the other symmetrical half of the instrument below the x-axis. Similarly, I can calculate the area enclosed by the interior perimeter of the violin. Since the inside of a violin is hollow with nothing inside, I will need to subtract the interior from the exterior to find the true wood area of the instrument. Correspondingly, I used Adobe Photoshop to visualize this process of integration. The "red" interior represents the hollow area of a violin. Subtracting it from the outlined "black" exterior area results in finding the "blue" border width of the instrument (exterior area - interior area = border width area). The "blue" width is the area that I aim to calculate.

Interior and exterior of a violin (Adobe Inc., 2024)



INTEGRATION OF THE MODERN VIOLIN

Before proceeding with integration, I found it beneficial to expand the equations for quadratic functions from the vertex form to the standard form ($ax^2 + bx + c$). This adjustment simplifies the integration process when performed by hand. To carry out the calculations, I relied on two fundamental rules: firstly, the antiderivative power rule allows me to undo the differentiation process, essentially determining the integral of a variable raised to an exponent. Secondly, the constant coefficient rule states that the coefficient of f(x) is a constant, which can be factored out and multiplied by the integral after applying the antiderivative power rule. Next, I will use definite integrals in terms of x because I chose to model the violin along the x-axis, which covers the range of x values, not y. To begin, I performed two calculations manually to cross-verify the results using a TI-84 Plus CE graphical display calculator. The calculator and my manual calculations resulted in the same answer. Thus, I continued to use the calculator for efficiency and to minimize potential errors (see Appendix A). I repeated this process for each point while carefully following the violin's perimeter that encloses the area of the violin.

<u>Calculations done by hand (exterior area)</u>

Point A to B

$$\int_0^{2.6}\left(\sqrt{28x}\right)dx$$

$$\int_0^{2.6}(28x)^{0.5}dx$$

$$=\left[\frac{(28x)^{1.5}}{1.5(28)}\right]_0^{2.6}-\left[\frac{(28x)^{1.5}}{1.5(28)}\right]_0^{2.6}$$

$$=\left[\frac{(28(2.6))^{1.5}}{1.5(28)}\right]_0^{2.6}-\left[\frac{(28(0))^{1.5}}{1.5(28)}\right]_0^{2.6}$$

$$=14.78930565cm^2$$

Point B to C

$$-0.07(x-7.65)^2+10.3$$

$$=-0.07x^2+1.071x+6.203425$$

$$\int_{2.6}^{12.2}\left(-0.07x^2+1.071x+6.203425\right)dx$$

$$\int_{2.6}^{12.2}-0.07x^2dx+\int_{2.6}^{12.2}1.071xdx+\int_{2.6}^{12.2}6.203425dx$$

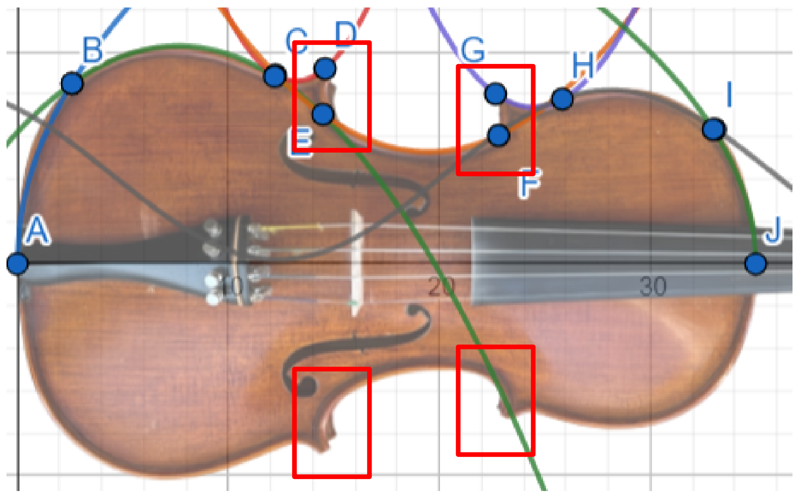
$$=\left[-\frac{0.07x^3}{3}\right]_{2.6}^{12.2}-\left[-\frac{0.07x^3}{3}\right]_{2.6}^{12.2}+\left[\frac{1.071x^2}{2}\right]_{2.6}^{12.2}-\left[\frac{1.071x^2}{2}\right]_{2.6}^{12.2}+[6.203425x]_{2.6}^{12.2}-[6.203425x]_{2.6}^{12.2}$$

$$=\left[-\frac{0.07(12.2)^3}{3}\right]_{2.6}^{12.2}-\left[-\frac{0.07(2.6)^3}{3}\right]_{2.6}^{12.2}+\left[\frac{1.071(12.2)^2}{2}\right]_{2.6}^{12.2}-\left[\frac{1.071(2.6)^2}{2}\right]_{2.6}^{12.2}+[6.203425\,(12.2)]_{2.6}^{12.2}-[6.203425\,(2.6)]_{2.6}^{12.2}$$

$$=93.67704cm^2$$

All the calculations were kept as exact values to ensure that before finding the density, there are no possibility of uncertainties, as it is more accurate than rounding to an approximate value. Bbetween points D to E and F to G, there is an overhang of ≈ 0.1cm, so I decided not to integrate those points as the overlap duplicates the area measured. In other words, integrating between the points C to D and G to H already accounts for the area under the overhang of the instrument. Next, I will subtract the interior from the exterior to find the border width area. As a result, I will first calculate the interior area of the violin. The border width of my violin is measured to be 0.2cm. Hence, I subtracted 0.2cm from each function, which is a translation of 0.2cm downwards; to calculate the interior hollow area, the border thickness of the violin will be removed.

Identifying the overhang of my modern violin (GeoGebra)



Calculations done by TI-84 Plus CE (interior area)

Point A to B

$$\int_0^{2.6} \left(\sqrt{28x} - 0.2\right) dx = 14.26930565 cm^2$$

Point B to C

$$\int_{2.6}^{12.2} \left(-0.07(x-7.65)^2 + 10.1\right) dx = 91.75704 cm^2$$

Point C to D

$$\int_{12.2}^{14.6} \left(0.3(x-13.2)^2 + 8.45\right) dx = 20.6544 cm^2$$

Point E to F

$$\int_{14.5}^{22.8} \left(0.07(x-19.5)^2 + 5.1\right) dx = 46.08519667 cm^2$$

Point G to H

$$\int_{22.7}^{25.8} \left(0.2(x-24.45)^2 + 7.2\right) dx = 22.84131667 cm^2$$

Point H to I

$$\int_{25.8}^{33.1} (4\sin(0.2(x+11.2)) + 4)\, dx = 54.86509581 cm^2$$

Point I to J

$$\int_{33.1}^{35} \left(\sqrt{-20(x-35)} - 0.2\right) dx = 7.428258045 cm^2$$

VOLUME AND DENSITY OF THE MODERN VIOLIN

Before finding the total volume of the modern violin, I need to add the areas of all the sets of points. Then, I will multiply the exterior area and interior area by 2, because the integration process only found the half area of the violin above the x-axis of the cartesian plane. Next, since the depth of the exterior and interior volumes are different, I will find both volumes separately. Using a ruler, I measured the depth of my violin: 3.8cm and because the border thickness is 0.2cm, the depth of the interior volume is 3.8 - 0.2 = 3.6cm. For the rest of the calculations, I rounded all answers to two decimal places, as a mere difference of 0.01cm can affect calculating the overall wood density and keep it consistent with price monetary values.

<u>Exterior area</u>: $2 \cdot (14.78930565 + 93.67704 + 21.1344 + 47.774519667 + 23.46131667 + 56.32509581 + 7.808258045) = 529.9398716\text{cm}^2$

<u>Exterior volume</u>: $529.9398716 \cdot 3.8 \approx$ **2,013.77cm^3**

<u>Interior area</u>: $2 \cdot (14.26930565 + 91.75704 + 20.6544 + 46.08519667 + 22.84131667 + 54.86509581 + 7.428258045) = 515.8012257\text{cm}^2$

<u>Interior volume</u>: $515.8012257 \cdot 3.6 \approx$ **1,856.88cm^3**

With this, I can subtract the interior volume from the exterior volume, which equals the true volume of the modern violin without its hollow centre.

<u>Overall volume</u>: $2{,}013.77 - 1{,}856.88 \approx$ **156.89cm^3**

To calculate the density (p) of the violin, I can divide the mass (m) by the volume (V) where $p = \frac{m}{V}$. First, I used an electronic luggage scale to measure the mass of my violin, which is 490 grams. However, the ebony (black wood) parts, including the fingerboard, tuning pegs, chinrest, and tailpiece, should be subtracted from 490g because it is heavy enough to affect the total mass of the instrument and do not account for the true mass of the violin wood. The non-ebony parts including the neck of the violin, bridge, strings, and fine tuners should be subtracted from 490g as they are also heavy, and are not part of the violin's core. Though, I do not have the tools to detach the ebony and non-ebony parts stuck to the core of my violin. Hence, I went to Amazon, which sells these violin parts, and found the mass of each component.

<u>Mass of ebony and additional parts</u>: fingerboard (120g) + tuning pegs (20g) + chinrest (40.8g) + tailpiece (19.8g) + neck (109g) + bridge (32g) + strings (9g) + fine tuners (20g) = 370.6g

<u>Mass of the violin</u>: 490 - 370.6 = **119.40g**

<u>Overall density</u>: 119.40 ÷ 156.89 ≈ **0.76g/cm^3**

After the calculations, I determined that the overall wood density of the modern violin is ≈ 0.76g/cm^3. Before evaluating the differences between both violins, I will first find the density of the Stradivarius violin, following a similar procedure as the modern violin, but with different physical properties and dimensions.

MODELLING THE STRADIVARIUS VIOLIN

To keep consistent with the modern violin's graphical representation, I used GeoGebra to model the graphs that surround the instrument. In the same way as the modern violin, I modelled the area of the Stradivarius violin. Then, I can use the function transformation rules accordingly. The length of the violin is 35.5cm (Violin Pros). Therefore, the graphs were transformed between the x values of 0 and 35.5 in centimeters.

$f(x) = -0.08(x - 7.75)^2 + 10.51$

$g(x) = \sqrt{35x}$

$h(x) = 0.14(x - 13.321)^2 + 8.856$

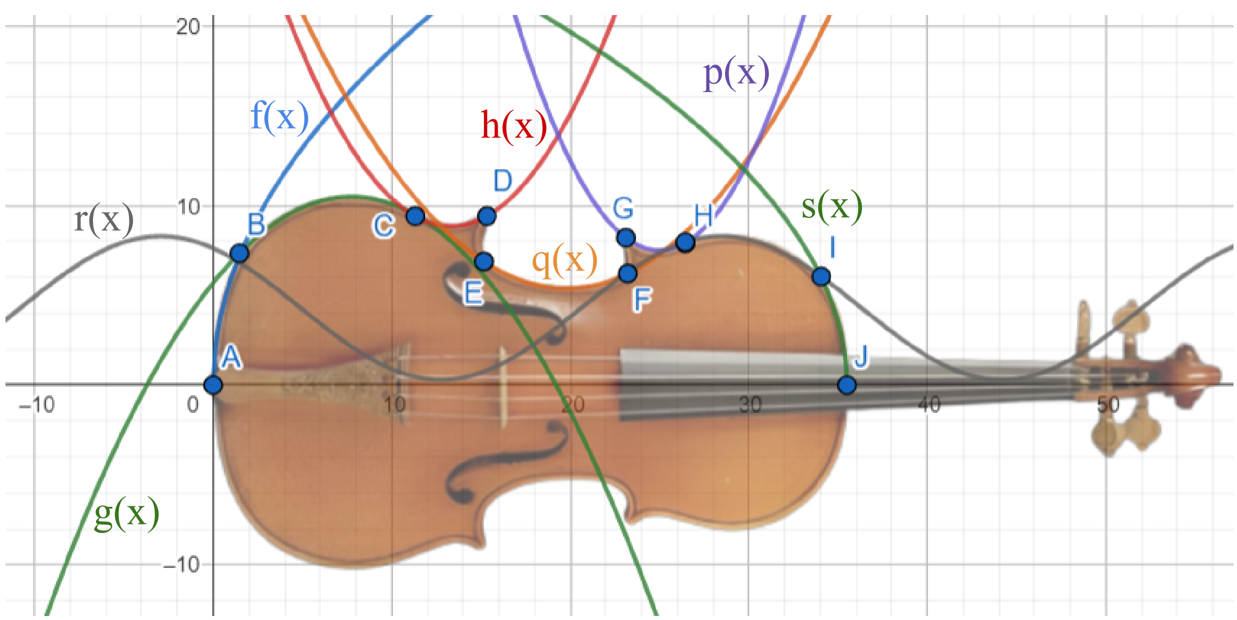
$q(x) = 0.07(x - 19.786)^2 + 5.397$

$p(x) = 0.2(x - 25)^2 + 7.5$

$r(x) = 4sin[0.2(x + 10.8)] + 4.3$

$s(x) = \sqrt{-25(x - 35.5)}$

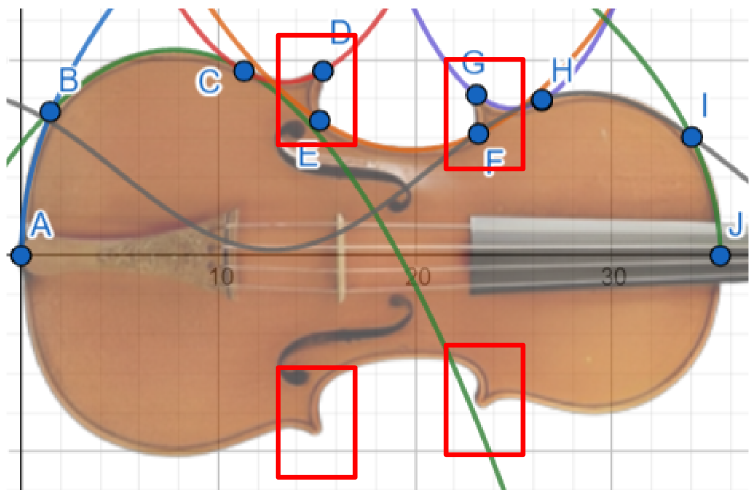
Modelling the area of the Stradivarius violin (GeoGebra)



INTEGRATION OF THE STRADIVARIUS VIOLIN

Since the process of finding the area of the modern and Stradivarius violin is repeated, I decided to separate the integration calculations into the appendix for conciseness. Between the points D to E and F to G, there is an overhang of 0.1cm similar to the modern violin. Therefore, I used the same value of 0.1cm and did not need to integrate between those points for the same reason as my modern violin. Accordingly, I will find the Stradivarius violin's wood border thickness, as the instrument is hollow and does not account for the true volume or density. The border thickness of a Stradivarius violin replica is 0.2 to 0.3cm (Violin Pros). Hence, I decided to use the average which equals 0.25cm, and subtract the value from each of the functions to calculate the hollow interior area of the violin. Specifically, this will vertically shift all the functions downwards from the original y coordinates. Altogether, for the modern violin and Stradivarius violin, I simply shifted all the functions downwards to find the interior area (see Appendix B).

Identifying the overhang of the Stradivarius violin (GeoGebra)



VOLUME AND DENSITY OF THE STRADIVARIUS VIOLIN

Following the exact procedure of the modern violin to find the density of the Stradivarius violin, I will first multiply the exterior and interior areas by 2. Next, the exterior and interior volumes will be found separately, as the depths for both volumes are different. I decided to use the modern violin's depth of 3.8cm for the depth of the Stradivarius violin because they are both full-size violins and have relatively the same dimensions. Accordingly, because the border thickness is 0.25cm, the depth of the interior volume is 3.8 - 0.25 = 3.55cm. For the rest of the calculations in this section, I rounded all answers to two decimal places to keep the measurements consistent with my prior calculations and to keep consistent with monetary values.

<u>Exterior area</u>: $2 \cdot (7.246 + 92.29 + 36.19473067 + 46.09953 + 25.3902 + 58.60357818 + 6.123724598) = 543.8955269\text{cm}^2$

<u>Exterior volume</u>: $543.8955269 \cdot 3.8 \approx$ **2,066.80cm^3**

<u>Interior area</u>: $2 \cdot (6.870688659 + 91.86805333 + 35.19473067 + 44.07453 + 24.5652 + 56.70357818 + 5.748724598) = 530.0510109\text{cm}^2$

<u>Interior volume</u>: $530.0510109 \cdot 3.55 \approx$ **1,881.68cm^3**

With this, I can subtract the interior volume from the exterior volume which equals the true volume of the Stradivarius violin without its hollow centre.

<u>Overall volume</u>: $2{,}066.80 - 1{,}881.68 \approx$ **185.12cm^3**

Then, I will use the formula $p = \frac{m}{V}$ to find the final density. The Messiah violin is unattainable because of its 20 million dollar valuation, and there are limited to no physical details of this classified instrument from the 18th century. Therefore, I recorded the mass of a Stradivarius violin replica, where the luthier tried to reproduce the exact dimensions and weight of the violin: 434 grams. Although the mass is not exact to the Messiah violin, it is close to what it should be. Likewise, with the modern violin, the ebony and additional parts should be subtracted from 434g because it is heavy enough to affect the total mass of the instrument. Correspondingly, I used the masses of the same ebony and additional parts of the modern violin, as it comes from the same type of wood.

<u>Mass of ebony and additional parts</u>: fingerboard (120g) + tuning pegs (20g) + chinrest (40.8g) + tailpiece (19.8g) + neck (109g) + bridge (32g) + strings (9g) + fine tuners (20g) = 370.6g

<u>Mass of the violin</u>: 434 - 370.6 = **63.40g**

<u>Overall density</u>: 63.40 ÷ 185.12 ≈ **0.34g/cm^3**

After the calculations, I determined that the overall wood density of the Stradivarius violin is ≈ 0.34g/cm^3. Forthwith, I can gather the wood density of the modern violin and Stradivarius violin to explore the differences between the two instruments. I can then investigate if the density of violin wood affects its valuation.

GRAPHICAL ANALYSIS

To answer my first question: if a higher or lower density instrument is more expensive, I can investigate if there is a correlation between wood density and price to determine whether one violin is more valuable than another. Firstly, I will record the data on various wood densities and their corresponding price. To keep the price of the wood relative to the density, I set the price to be for every cubic centimeter. Earlier, I calculated the total volume of each violin; I can divide the violin's initial cost by the volume to find the price in United States dollars, $USD per cm^3. Accordingly, I will research every other wood type online to find their densities and prices; since beginner violins are not the best quality instruments compared to the modern and Stradivarius violin, my research is based on the prices of professional violins made by credible luthiers or brands. Because I already have the volume of the modern and Stradivarius violin, I, therefore, decided to use the mean volume $(156.89 + 185.12) \div 2 \approx 171.01\text{cm}^3$ as the representation for a typical violin, with dimensions in between a cheaper and more expensive violin. Although this process may encourage systematic errors due to the nature of violin styles that can have countless shapes and sizes; it is tedious to remodel numerous types of violins repeatedly to find the volumes. I will then divide each wood price by the volume of 171.01cm^3 to find the unitary price per cubic centimeter. The process will be done for the most common types of wood that make a playable violin (see Appendix C). Lastly, the values are rounded to two decimal places to keep the price monetary values consistent.

<u>Modern Violin Wood Price</u>: $\$1{,}000 \div 156.89\text{cm}^3 \approx$ **$6.37USD/cm^3**

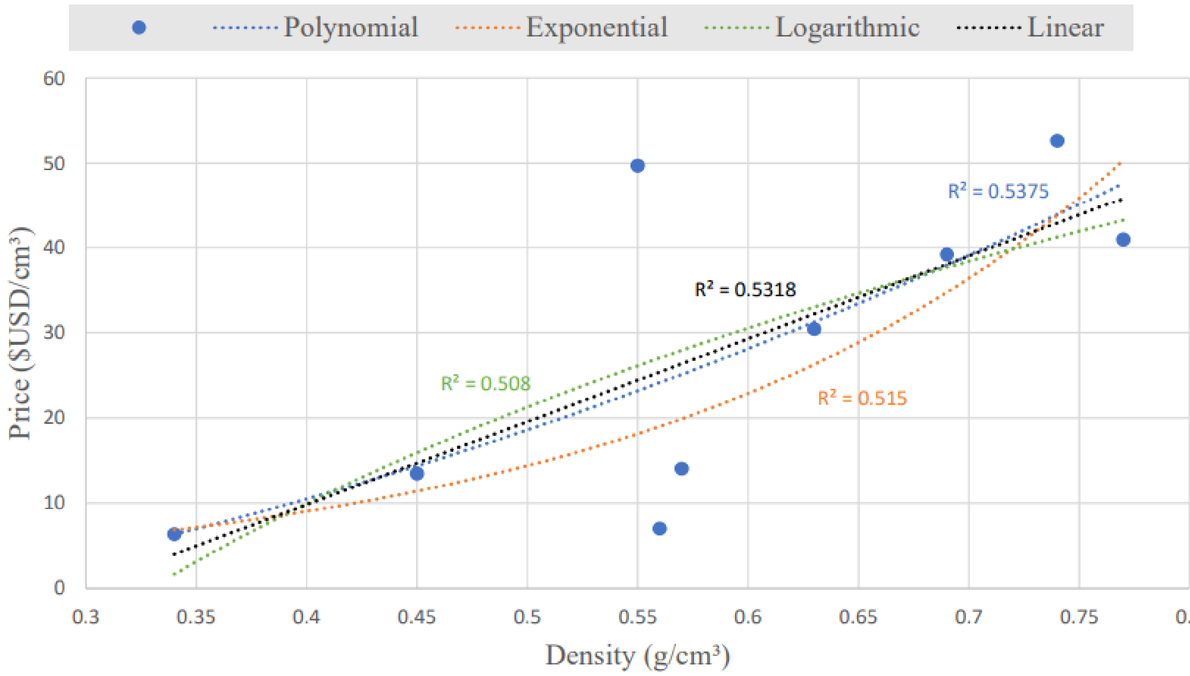
<u>Stradivarius Violin Wood Price</u>: $\$20{,}000{,}000 \div 185.12\text{cm}^3 \approx$ **$108,038.30/cm^3**

Table for violin density versus price (Engineering Toolbox, 2004)

Type of Wood	Density (g/cm^3)	Price ($\$USD/cm^3$)
Modern Violin	0.34	6.37
Spruce	0.45	13.45
Sycamore	0.55	49.71
Elm Dutch	0.56	7.02
Spruce and Maple Mixed	0.57	14.03
Cherry European	0.63	30.41
Maple	0.69	39.18
Oak American Red	0.74	52.63
Stradivarius Violin	0.76	108,038.30
Oak American White	0.77	40.93

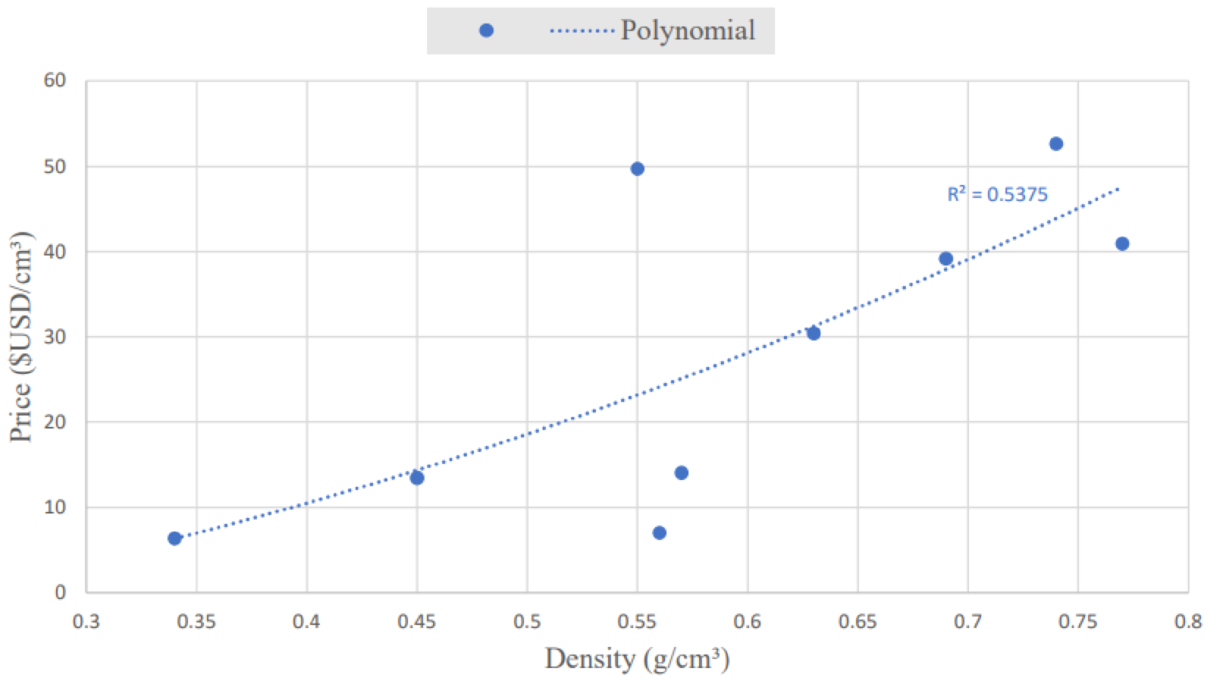
Observing the data points, the unitary price for the Stradivarius violin does not help with investigating my aim, because it is an outlier costing a drastic hundred thousand more $\$USD/cm^3$ than any other type of wood. This fact suggests that the Stradivarius violin is an outlier for historical factors; because authentic Stradivarius violins are no longer handcrafted in Italy since 1737, it makes the instrument much more expensive than any other standard violin in the market (Stradivarius). Next, it is evident that the final density I calculated for the modern violin is accurate as it is close to all the densities of other types of wood. At the same time, the density is lower than every type of wood, which indicates that there may be gaps in the model of my graph and does not guarantee the complete accuracy of its area, volume, and density. Otherwise, I can graph the data points and remove the Stradivarius violin outlier to be able to examine the graph within a closer range.

Density vs. Price Graph Figure 1 (Microsoft Corporation, 2018)



As illustrated in Figure 1, the regression curves that I used to examine the data points were degree 2 polynomial (quadratic), exponential, logarithmic, and linear functions. Examining the scatter plot I made with density as the independent variable x, and price as the dependent variable y, I noticed that each modelled regression has an $R^2 > 0.5$ which shows that in more than 50% of cases, I can extrapolate that a higher wood density of a violin means a higher price and vice versa. However, I gathered all the R^2 values from the trend line functions and I chose to use the quadratic with a coefficient of determination of 0.5375; the regression predictions are the most accurate with the R^2 closest to a perfect 1. Consequently, I removed the R^2 functions that are not needed to analyze the data points and drew a new graph with only the quadratic function for the regression model.

<u>Density vs. Price Graph Figure 2 (Microsoft Corporation, 2018)</u>



A weak R^2 value is below 0.5 with less than 50% of data points not following the regression. On the other hand, an R^2 value above 0.9 is considered a strong regression model as nearly 100% of data points follow the trend line. Although the coefficient of determination from Figure 2 is the most accurate compared to the others, around 50% of cases are variabilities in the outcome of data, and the regression line does not follow some of the data points. Similar to the Stradivarius violin, some types of wood may be inconsistent due to valuable historical elements or its rarity, which can affect the overall price in the real world. Predominantly, the model follows a slight upward curve of a parabolic function, suggesting that by the quadratic regression, there is a nonlinear relationship between the wood density of a violin and its price. With discrepancies in the model, it is unclear to justify a relationship between the density and the price of the wood of a violin.

CONCLUSION

Does the density of a violin affect the price? To answer my aim, I discovered that there is no correlation between the two variables, and the density of a violin does not influence the instrument's price. Firstly, I excluded the data retrieved from the outlier Stradivarius violin, which was a central part of my investigation. Moreover, the regression model between the density and price of a violin does not justify a quadratic relationship between the two variables. This is because the data points were retrieved from raw wood instead of violin wood. The used densities of the raw wood species have fixed prices that differ from the handmade stylistic appearances of a violin. As shown with the Stradivarius violin, the wood price can also be affected by historical factors and the subjective perspective of consumers believing what is cheap or valuable. These factors can be said the same with any other violin in the market, which induces drastic price differences such as with the 20 million dollar Stradivarius violin. Consequently, this exploration reveals that the density of a violin does not affect its price.

EVALUATION

There were a few strengths in my study despite not finding an accurate correlation between the two variables. For instance, there was no trial and error during the process of integrating the violins' outlines. My procedure did not involve advanced technology and equipment without additional expenses along with using my personal violin. This exploration allowed me to highlight the musical instrument I love to play the most and provides concrete evidence that the density of a musical violin does not affect its price. However, there are extraneous variables that could have affected my calculations during the process, which affects the validity of my results. For instance, my exploration was purely done from a two-dimensional perspective. This limits finding the exact three-dimensional shape, as the depth of a violin may have imperfections that do not make the instrument perfectly symmetrical. Some limitations are also apparent as the violin has many non-detachable components such as the fingerboard, tuning pegs and strings of the violin that significantly affect its overall mass. With this, I was not able to measure the exact mass of the extra ebony parts which may have affected the degree of accuracy of the final density for both the modern and Stradivarius violins. Lastly, my method of removing the border thickness to find the interior area of the instrument is not fully accurate, because it does not allow the area from the x coordinates to be removed, only y. Although I could have transformed the functions to shrink the interior area in both the x and y directions, it does not let me precisely measure an exact value to be removed from every angle. In the future, I can apply my findings to other wood instruments to justify if the non-correlation between the price and density of a violin is the same with guitars, pianos, drums, harps, etc. Using this investigation, I can extend my application of calculus, specifically integration to a three-dimensional shape of an instrument, which can help myself and others with engineering, printing, or hand making new instrumental designs.

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Appendix A

Integration of the Modern Violin

I manually integrated the first two calculations to ensure that my calculator got the same answer. Then, as they were equal to each other, I continued the process with my calculator to maintain efficiency, rather than integrating step-by-step manually.

<u>Calculations done by TI-84 Plus CE (exterior area)</u>

Point C to D
$$\int_{12.2}^{14.6} \left(0.3(x-13.2)^2+8.65\right)dx$$
$$= 21.1344cm^2$$

Point E to F
$$\int_{14.5}^{22.8} \left(0.07(x-19.5)^2+5.3\right)dx$$
$$= 47.774519667cm^2$$

Point G to H
$$\int_{22.7}^{25.8} \left(0.2(x-24.45)^2+7.4\right)dx$$
$$= 23.46131667cm^2$$

Point H to I
$$\int_{25.8}^{33.1} (4\sin(0.2\,(x+11.2))+4.2)\,dx$$
$$= 56.32509581cm^2$$

Point I to J
$$\int_{33.1}^{35} \left(\sqrt{-20\,(x-35)}\right)dx$$
$$= 7.808258045cm^2$$

Appendix B

Integration of the Stradivarius Violin

Integrating the area of the Stradivarius violin is the same as my modern violin with different values. Therefore, I pasted the calculations into the appendix to avoid repetition in my work. I manually integrated the first two calculations to ensure that my calculator got the same answer. Then, as they were equal to each other, I continued the process with my calculator to maintain efficiency, rather than spending extra time integrating step-by-step by hand. First, I calculated the exterior area of the Stradivarius violin. Then, I subtracted 0.25cm from the wood border thickness, to find the interior area of the violin (vertical shift downwards of 0.25 for every function).

<u>Calculations done by hand (exterior area)</u>

Point A to B

$\int_0^{1.5} \left(\sqrt{35x}\right) dx$

$\int_0^{1.5} (35x)^{0.5} dx$

$= \left[\frac{(35x)^{1.5}}{1.5(35)}\right]_0^{1.5} - \left[\frac{(35x)^{1.5}}{1.5(35)}\right]_0^{1.5}$

$= \left[\frac{(35(1.5))^{1.5}}{1.5(35)}\right]_0^{1.5} - \left[\frac{(35(0))^{1.5}}{1.5(35)}\right]_0^{1.5}$

$= 7.245688659cm^2$

Point B to C

$-0.08(x - 7.75)^2 + 10.505$

$= -0.08x^2 + 1.24x + 5.7$

$\int_{1.5}^{11.2} \left(-0.08x^2 + 1.24x + 5.7\right) dx$

$\int_{1.5}^{11.2} -0.08x^2 dx + \int_{1.5}^{11.2} 1.24x dx + \int_{1.5}^{11.2} 5.7dx$

$= \left[-\frac{0.08x^3}{3}\right]_{1.5}^{11.2} - \left[-\frac{0.08x^3}{3}\right]_{1.5}^{11.2} + \left[\frac{1.24x^2}{2}\right]_{1.5}^{11.2} - \left[\frac{1.24x^2}{2}\right]_{1.5}^{11.2} + [5.7x]_{1.5}^{11.2} - [5.7x]_{1.5}^{11.2}$

$= \left[-\frac{0.08(11.2)^3}{3}\right]_{1.5}^{11.2} - \left[-\frac{0.08(1.5)^3}{3}\right]_{1.5}^{11.2} + \left[\frac{1.24(11.2)^2}{2}\right]_{1.5}^{11.2} - \left[\frac{1.24(1.5)^2}{2}\right]_{1.5}^{11.2} + [5.7\,(11.2)]_{1.5}^{11.2} - [5.7\,(1.5)]_{1.5}^{11.2}$

$= 94.29305333cm^2$

Calculations done by TI-84 Plus CE (exterior area)

Point C to D
$\int_{11.2}^{15.2} \left(0.14(x-13.32)^2+8.86\right) dx$
$= 36.19473067cm^2$

Point E to F
$\int_{15.1}^{23.2} \left(0.07(x-19.5)^2+5.3\right) dx$
$= 46.09953cm^2$

Point G to H
$\int_{23.1}^{26.4} \left(0.2(x-25)^2+7.5\right) dx$
$= 25.3902cm^2$

Point H to I
$\int_{26.4}^{34} (4\sin(0.2\,(x+10.8))+4.3)\, dx$
$= 58.60357818cm^2$

Point I to J
$\int_{34}^{35.5} \left(\sqrt{-25\,(x-35.5)}\right) dx$
$= 6.123724598cm^2$

Calculations done by TI-84 Plus CE (interior area)

Point A to B
$\int_{0}^{1.5} \left(\sqrt{35x}-0.25\right) dx$
$= 6.870688659cm^2$

Point B to C
$\int_{1.5}^{11.2} \left(-0.08(x-7.75)^2+10.255\right) dx$
$= 91.86805333cm^2$

Point C to D
$\int_{11.2}^{15.2} \left(0.14(x-13.32)^2+8.61\right) dx$
$= 35.19473067cm^2$

Point E to F
$\int_{15.1}^{23.2} \left(0.07(x-19.5)^2+5.05\right) dx$
$= 44.07453cm^2$

Point G to H
$\int_{23.1}^{26.4} \left(0.2(x-25)^2+7.25\right) dx$
$= 24.5652cm^2$

Point H to I
$\int_{26.4}^{34} (4\sin(0.2\,(x+10.8))+4.05)\, dx$
$= 56.70357818cm^2$

Point I to J
$\int_{34}^{35.5} \left(\sqrt{-25\,(x-35.5)}-0.25\right) dx$
$= 5.748724598cm^2$

Appendix C

Finding the Unitary Price for Types of Wood

Elm Dutch Wood Price: \$1,200 $\div$ 171.01cm^3 $\approx$ **\$7.02USD/cm^3**

Spruce Wood Price: \$2,300 $\div$ 171.01cm^3 $\approx$ **\$13.45USD/cm^3**

Spruce and Maple Mixed Wood Price: \$2,400 $\div$ 171.01cm^3 $\approx$ **\$14.03USD/cm^3**

Cherry European Wood Price: \$5,200 $\div$ 171.01cm^3 $\approx$ **\$30.41USD/cm^3**

Maple Wood Price: \$6,700 $\div$ 171.01cm^3 $\approx$ **\$39.18USD/cm^3**

Oak American White Wood Price: \$7,000 $\div$ 171.01cm^3 $\approx$ **\$40.93USD/cm^3**

Sycamore Wood Price: \$8,500 $\div$ 171.01cm^3 $\approx$ **\$49.71USD/cm^3**

Oak American Red Wood Price: \$9,000 $\div$ 171.01cm^3 $\approx$ **\$52.63USD/cm^3**