

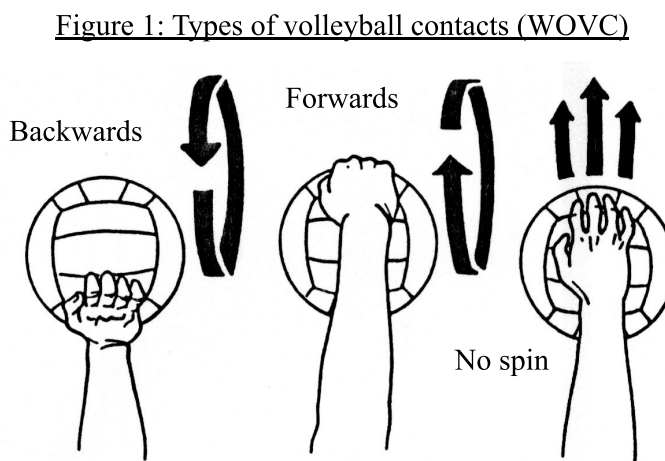
TO EXPLORE IF THE VELOCITY OF A VOLLEYBALL SERVE AFFECTS THE ANGULAR VELOCITY OF THE BALL TO A FIXED TARGET

Introduction

Since I was a kid, I was always attached to playing various kinds of sports. For instance, water, combat, track and field, target, ball, and racquet sports. However, the sport that stuck with me the most was volleyball. Since I was 12 years old, I played volleyball consistently in clubs and school teams. I learned to be a consistent server for many years and got the ball over the net for my team when needed. However, I never got the most optimal serve to get a point for my team after every set. In the Olympics and seasonal leagues, professional players are known to make serves that curve, serves that are made for speed, and power. As a kid, I was always intrigued by how a ball can be manipulated mid-air in countless possible ways, and what causes this observable phenomenon. In class, I briefly learned about the Magnus effect and how it interconnects with projectile motion in the topic of kinematics. I was fascinated by how a lift force can act on a spinning object which can deflect and significantly manipulate the path of a volleyball serve. Accordingly, I wondered if a higher or lower velocity results in the need for more spin? And how is the Magnus effect force significant when a volleyball changes in the speed of the spin? My aim is thus, **to explore if the velocity of a volleyball serve affects the angular velocity of the ball to a fixed target.**

Background Information

It is important to consider air resistance as it is always present on this planet. Although it may be imperceptible, the air has a mass and significantly affects the final displacement of a volleyball serve. The air's density with the Magnus effect impacts the projectile of a volleyball serve (The Magnus Project). I can use my hand to manipulate the hit contact of the volleyball, which changes the target of the serve. This exemplifies the practicality of kinematics and fluid dynamics in real-world conditions and reveals the observable phenomenon of the Magnus effect. Based on the type of initial contact on a ball, a volleyball serve can be manipulated freely. However, from experience and in gameplay, it is easier to apply a forward rotation on a ball. For this reason, I will study forward contact rather than backwards or still rotation.



Correspondingly, I hypothesize that **an increased forward rotation (angular velocity) applied on a volleyball serve will result in the need to decrease its velocity to reach a fixed target and vice versa.**

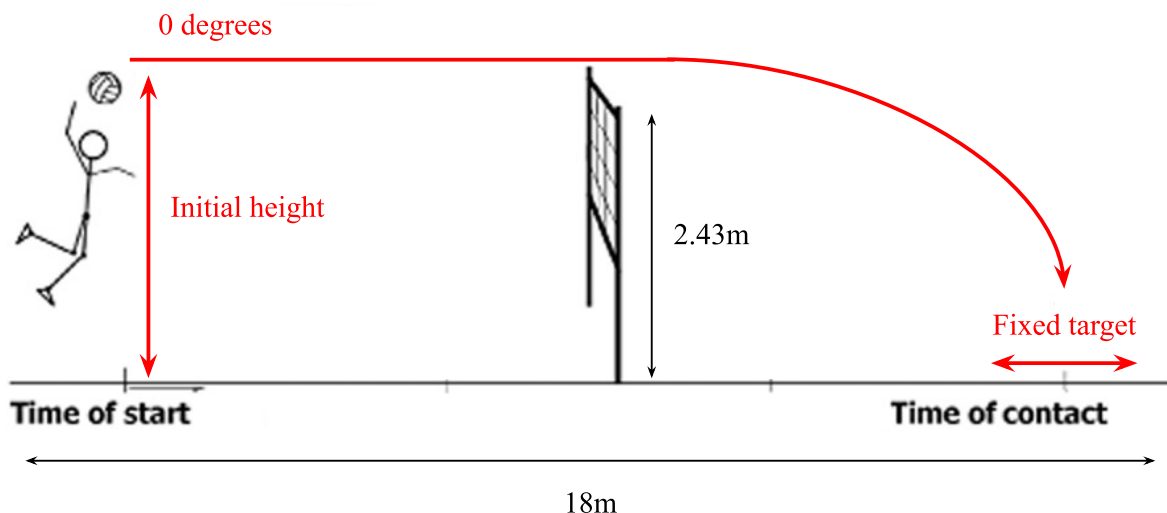
Apparatus

- Volleyball
- Indoor court
- Adjustable volleyball net
- iPhone camera

Methodology

1. Measure the distance of the full length of a volleyball court.
2. Determine the angle in degrees at which I will be serving the ball.
3. Measure the height of my vertical jump in centimetres (when serving) to set a constant initial height of the serve.
4. Acquire a slow-motion camera to measure the velocity and speed of the spin of the volleyball.
5. Establish five different types of serve contacts for five trials.
6. Use a standard 21cm diameter volleyball, and do five jump spin serves over a net for every trial; record the data.
7. Graph the uncertainties and plot the averages for velocity versus angular velocity.
8. I will examine the changes in velocity, versus the changes in the speed of the spin.
9. I will examine how the Magnus effect impacts the projectile motion of the volleyball.
10. I will examine the amount of time taken to reach the ground.

Figure 2: Projectile motion of volleyball Serve (Benerink et al., 2015)



Safety Precautions

The main risk of the investigation is hitting the volleyball at bystanders or other volleyball players on the court. Resultantly, to avoid potential injuries, particularly brain trauma, including concussions, the experiment will be conducted in a primarily empty volleyball court to ensure that no one is in the way of the flight path of the serve. Additionally, I performed a visual scan around the court before serving to ensure the safety of others in the gymnasium.

When holding down the wires and gears to set the volleyball net in the middle of the court, some hazards are common for injury. The poles on opposite sides of a volleyball net are heavy and it is vital to ensure my safety when carrying the parts to set the net by avoiding tripping or slipping on the court.

Uncertainties

To begin, I will need to account for the possible systematic errors and uncertainties. As it is practically impossible to stop a slow-motion video at the exact point every time, the uncertainty in time is ± 0.1 s. It is also impossible to toss a volleyball at the same height before hitting the serve, therefore there is an uncertainty of ± 0.1 m to the height. To add on, the x-displacement will have an uncertainty ± 1 m because the ball will land on a fixed target with a radius of 1m. A volleyball never lands on an exact spot whenever served by a human and will not hit perfectly at the center of the fixed target. Finally, I used an increment of 45 degrees to measure the total rotations of the ball systematically. I did not use the radians, because degrees are easier to visualize from a camera screen. There may be misinterpretations of the total revolutions of the ball due to technological errors by ± 45 degrees.

Variables

Before beginning to collect my data, I will need to establish the control variables of this experiment to keep consistency, and avoid trial and error:

- Length of the volleyball court, 18 ± 1 m
- Height of my vertical jump, 2.75 ± 0.1 m
- Initial angle of the serve is parallel to the ground, 0 degrees
- Length of the fixed target for the ball to land on the ground, 2m

I gathered the variables for the aim of this exploration and set velocity as the independent variable. The angular velocity is my dependent variable because the goal is to investigate if the velocity of a volleyball impacts the result of needing to spin the ball to get it to a fixed target.

Data Collection

The total x-displacement of the volleyball serve will be $18 \pm 1\text{m}$ because throughout my life playing and watching volleyball, I learned that the most optimal serves land at the very end of the court. My 5 trials will be separated by the various types of volleyball serves that are controlled using my contact hand, each manipulated with a different speed. The independent raw data is the time taken (t) in seconds (s) before the volleyball touches the ground. To process the data and calculate the independent variable velocity (v_x) in meters per second (ms^{-1}) I will take $x\text{-displacement} \div \text{time}$. The dependent raw data is the total degrees rotated by the ball ($^\circ$) before reaching the ground. To process the data and calculate the dependent variable angular velocity (ω) in revolutions per second (rs^{-1}) I will take $\text{degrees turned} \div 360 \div \text{time}$.

Lastly, the experimental errors will be calculated. The raw data will not be fully rounded to ensure precision when processing the data. For instance, time will be rounded to two significant figures as the absolute uncertainties are less than one and it is important for two decimal places to keep precision for future calculations. Similarly, I will round the degrees turned to the nearest whole number to visualize precisely the amount of degrees turned in the total of 360° . However, there will be no decimal significant figures, as it is relatively insignificant for the amount turned for one whole or 360° rotation. For the processed data, rules for significant figures will be applied excluding the control variables (as it is constant and does not contribute to the precision of the final result). The uncertainties for the processed data will be rounded to one significant figure to simplify the math and minimize the propagation of unnecessary precision throughout the calculations.

Sample Calculations With Uncertainties - Trial #1

t (s)	v_x (ms^{-1})
$= 2.34 \pm \frac{\text{max} - \text{min}}{2}$ $= 2.34 \pm \frac{2.42 - 2.22}{2}$ $= 2.34 \pm 0.10\text{s}$	$= x\text{-displacement} \div t$ $= (18 \pm 1) \div (2.34 \pm 0.1)$ $= 7.7 \pm \left(\frac{1}{18} + \frac{0.1}{2.34}\right) \cdot 7.70$ $= 7.7 \pm 0.8\text{ms}^{-1}$
degrees turned ($^\circ$)	ω (rs^{-1})
$= 342 \pm \frac{\text{max} - \text{min}}{2}$ $= 342 \pm \frac{450 - 225}{2}$ $= 342 \pm 110^\circ$	$= \frac{\text{degrees turned} \div 360}{t}$ $= \frac{(342 \pm 110) \div 360}{2.34 \pm 0.1}$ $= 0.41 \pm \left(\frac{110 \div 360}{342 \div 360} + \frac{0.1}{2.34}\right) \cdot 0.41$ $= 0.4 \pm 0.2\text{rs}^{-1}$

Raw to Processed Data Calculations (see Appendix A)

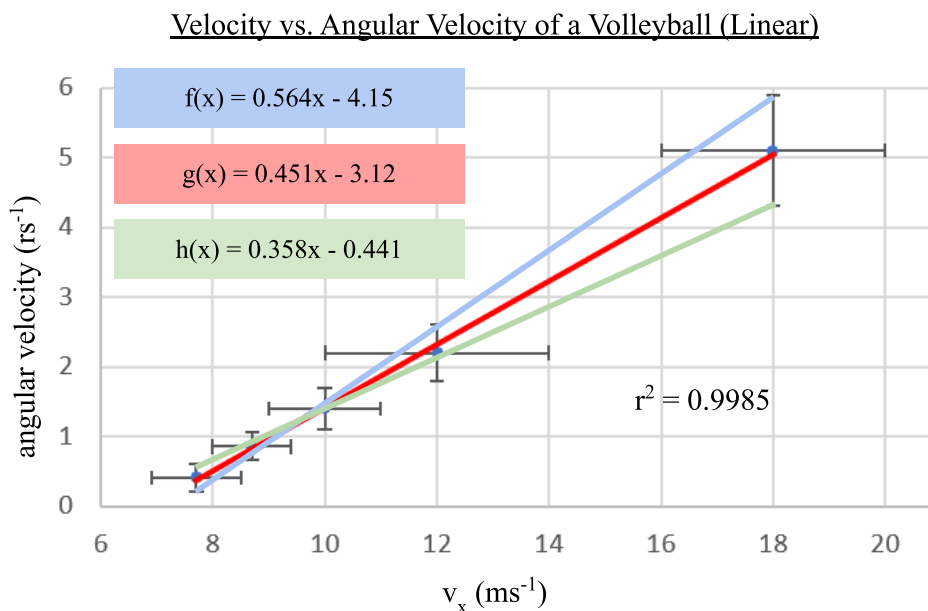
TRIAL #	time (s) ± 0.1	v_x (ms ⁻¹)	degrees turned ($^\circ$) ± 45	ω (rs ⁻¹)
1 Initial contact without spin	2.40		270	
	2.32		360	
	2.34		405	
	2.42		225	
	2.22		450	
Mean:	2.34 $\pm$ 0.10	7.7 $\pm$ 0.8	342 $\pm$ 110	0.4 $\pm$ 0.2
2 Toss the ball with spin, initial contact without spin	2.04		720	
	2.12		540	
	2.08		495	
	2.04		765	
	2.00		675	
Mean:	2.06 $\pm$ 0.06	8.7 $\pm$ 0.7	639 $\pm$ 135	0.9 $\pm$ 0.2
3 Toss the ball without spin, initial contact with spin	1.62		945	
	1.84		990	
	1.74		900	
	1.68		765	
	1.74		720	
Mean:	1.72 $\pm$ 0.11	10 $\pm$ 1	864 $\pm$ 135	1.4 $\pm$ 0.3
4 Toss the ball with spin, initial contact with spin	1.64		1125	
	1.54		1215	
	1.40		1260	
	1.40		1080	
	1.46		1350	
Mean:	1.50 $\pm$ 0.12	12 $\pm$ 2	1206 $\pm$ 135	2.2 $\pm$ 0.4
5 Maximum power	1.00		1800	
	1.04		1800	
	1.06		1755	
	0.98		1715	
	0.92		2025	
Mean:	1.00 $\pm$ 0.07	18 $\pm$ 2	1819 $\pm$ 155	5.1 $\pm$ 0.8

Graphical Analysis

To model the velocity of the ball versus the need to spin the ball. I will use “ v_x ” to represent the velocity in the x direction. I used Excel to organize my data and calculated the mean for each of the five trials. Afterwards, I will create a scatter plot to investigate if there is a proper line of best fit to examine if there is a correlation.

Data Points

v_x (ms^{-1})	ω (rs^{-1})
7.7 ± 0.8 or 10%	0.4 ± 0.2 or 50%
8.7 ± 0.7 or 8%	0.9 ± 0.2 or 22%
10 ± 1 or 10%	1.4 ± 0.3 or 21%
12 ± 2 or 17%	2.2 ± 0.4 or 18%
18 ± 2 or 11%	5.1 ± 0.8 or 16%



It is clear when the v_x increases, the rs^{-1} has to increase as well. However, as both variables increase along the straight line of best fit, the uncertainties increase and suggest that it is more difficult to track a volleyball serve when it has a higher velocity. The correlation coefficient r^2 is 0.9985, which shows a high degree of precision in the data. However, the data still lacks accuracy and may present random errors as indicated by the vertical and horizontal error bars.

Hence, I can calculate the uncertainty in the gradient, when $y = ax + b$ to reflect the presence of error in my data.

Absolute uncertainty in the gradient

(difference of maximum gradient and minimum gradient, divided by 2):

$$\Delta m = \frac{\Delta y}{\Delta x}$$

$$\Delta m = \frac{0.564 - 0.358}{2}$$

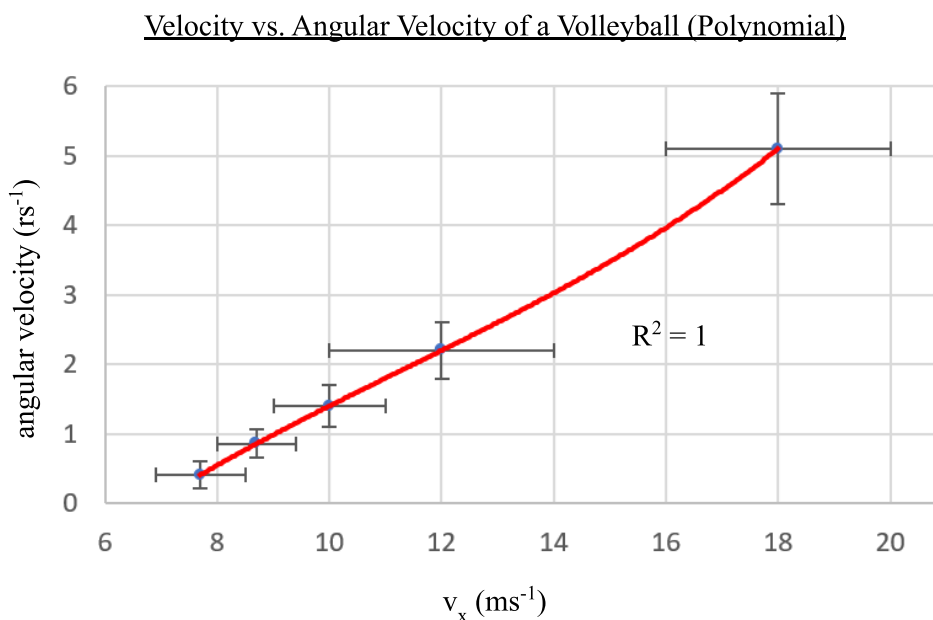
$$\Delta m = \pm 0.103$$

Percentage uncertainty in the gradient

(absolute uncertainty, divided by the gradient of the middle curve):

$$\frac{0.103}{0.451} \cdot 100 \approx \pm 22.8\%$$

22.8% is significant for uncertainty and may be reflective of variabilities. Therefore, I experimented with different regressions and determined that a degree 3 polynomial is the optimal line of best fit.



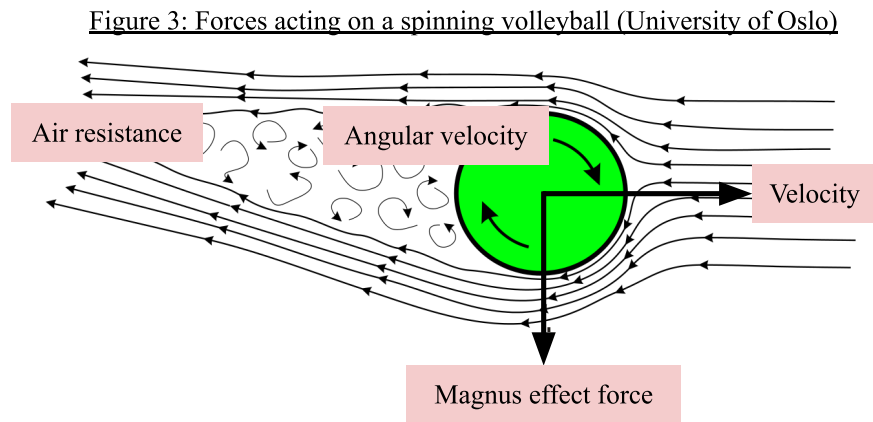
As shown, the R^2 value of 1 is perfect and is statistically considered to be a better fit for the data than the linear model. Hence, the degree 3 polynomial represents an inflection point when velocity reaches 14ms^{-1} and shows that values beyond 14ms^{-1} will require the angular velocity to drastically increase compared to velocity below 14ms^{-1} .

Magnus Effect

To continue this investigation, I wondered: how does the Magnus effect relate to the need to spin the ball when velocity is increased? All objects on the Earth encounter air resistance except those in a vacuum. With air resistance, there are changes in the magnitude of the spin of a volleyball and induces a Magnus effect force that is perpendicular to its velocity. I have encountered countless variations of

volleyball serves in gameplay, however, the serves with a downward curve are the most difficult to pass. This downward curve is highly deceptive and will change a spinning ball to move faster towards the ground making it troublesome for a volleyball player to reach

the ball. Therefore, it is important to explore the differences in this perpendicular force to answer the aim of this exploration and how it affects the need to spin the ball depending on the changes in velocity to hit a fixed target. For the following calculations, I used the Magnus effect formula:



Magnus effect force: $F_m = 2\pi S \omega v_x$ where, S = air resistance, ω = angular velocity, v_x = velocity in the x direction (The Magnus Project). The International System (SI) of Unit for angular velocity is in radians per second, therefore I will have to convert by multiplying the total rotations of the ball by 2π representing one revolution. Then, I will need to calculate the air resistance:

Air resistance: $S = \frac{1}{2} \rho v_x^2 C A$ where, ρ = density of air, v_x = velocity in the x direction, C = drag coefficient, A = two-dimensional area of the ball. The last unknown variable is the drag coefficient and I can simply rearrange the previous equation. I can measure my volleyball to find the two-dimensional area of the ball (A) from a bird's-eye view, and I can research the standard value of the density of air (ρ). Therefore, $A = 0.035\text{m}^2$ and according to the International Standard Atmosphere (ISA) $\rho = 1.23\text{kg/m}^3$ (Toppr).

Lastly, I decided to use 0.17 for the drag coefficient of a standard Molten volleyball, as I do not have the technology to find the value myself (Asai et al., 2010). Before finding the Magnus effect force, there is an unknown variable: air resistance. For each trial, I will solve for air resistance (S) to then find the Magnus effect force (F_m) both in newtons (N). I calculated for the first trial below and inserted the remaining calculations in the appendix to avoid repetition.

Trial #1

$$S = \frac{1}{2} \rho v_x^2 C A$$

$$S = \frac{1}{2} \cdot 1.23 \cdot 7.7^2 \cdot 0.17 \cdot 0.035$$

$$S = 0.22 \text{ N}$$

$$F_m = 2\pi S \omega v_x$$

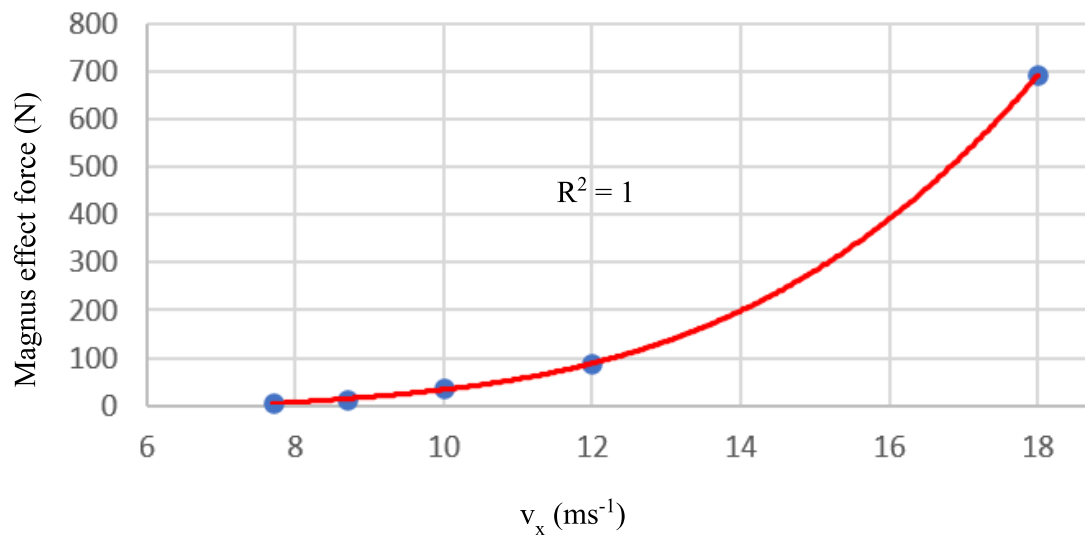
$$F_m = 2\pi \cdot 0.22 \cdot 0.41 \cdot 7.7$$

$$F_m \approx 4.4 \text{ N}$$

Uncertainties are neglected as they are not needed for the remainder of this investigation and to focus better on the scientific proof of why the angular velocity is affected by changes in velocity.

Magnus Effect Force - Results (see Appendix B)

TRIAL #	$v_x \text{ (ms}^{-1}\text{)}$	S (N)	$F_m \text{ (N)}$
1	7.7	0.33	4.4
2	8.7	0.38	13
3	10	0.37	35
4	12	0.53	88
5	18	1.2	690

Velocity vs. Magnus Effect Force

Air resistance changes relative to when the velocity changes. Similarly, the angular velocity of the ball drastically impacts the Magnus effect force that directs the ball to a specific location. As shown on the graph, $R^2 = 1$ represents a perfect model, proving that there is a third-degree polynomial relationship as the velocity increases, the Magnus effect force significantly increases.

The reason why the angular velocity of a volleyball must increase as velocity increases is because of the Magnus effect. Without this phenomenon, spinning a volleyball would have no purpose in gameplay and the ball would be impossible to be curved.

Conclusion

Does the velocity of a volleyball serve affect the angular velocity of the ball to a fixed target? To answer my aim, I discovered that there is an increasing relationship between the two variables and the velocity of a volleyball serve does eminently affect the angular velocity of a fixed target. As velocity increases, the angular velocity increases to ensure the ball lands in the target, and vice versa. Adding on, the visible trend is due to the Magnus effect force and its nature of a downward perpendicular thrust to bring the volleyball down to the fixed target. In conclusion, this exploration revealed a discernible relationship between the data of the two variables I collected. Gathering my information, I was provided with valuable insights into the dynamics of the projectile motion of a volleyball specifically with the impact of air resistance from the Magnus effect. Not only does it provide theoretical knowledge in physics, but it highlights the practical significance of these variables when optimizing the performance in volleyball gameplay. Overall, I was presented with data that adds a valuable layer to the multifaceted nature of kinematics that can be applied to countless other sports in the world.

Evaluation

Strengths

There were a few strengths in my study that allowed me to highlight the sport I love the most.

1. My results were consistent with my hypothesis, where velocity undeniably affects the angular velocity to manipulate the final displacement of the volleyball.
2. My procedure did not involve advanced technology and equipment without additional expenses, while I was still able to accomplish my aim. For instance, instead of relying on bulky and expensive equipment to measure the drag coefficient of the volleyball, I used the internet to research an approximate value of a standard volleyball.

3. I was successful in finding an accurate relationship between the two variables without trial and error. This minimized the random errors of my calculations as the slow-motion camera ensured to accurately capture the motion of the volleyball serve.
4. Both polynomial regression models follow a trend line with an $R^2 = 1$. This indicates that the regression predictions and points perfectly fit the data. The graph comparing the velocity versus the angular velocity, strongly suggests that there is a relationship between the two variables. Correspondingly, the graph comparing the velocity versus Magnus effect force follows a similar trend line. This indicates that my models are consistent and are successful in providing the answer to my research question.

Weaknesses

My exploration had significant weaknesses that may have hindered the final results and findings.

1. There were extraneous variables that could have affected my calculations during the process, which affected the validity of my results. For instance, the volleyball may have slightly deflated throughout my experiment and after every serve, which affects the Magnus effect force and the surface area of the ball being influenced by air resistance.
2. My investigation was purely done from a two-dimensional perspective and I did not account for variables, such as velocity in a third axis that can impact the final displacement of the ball. The world is perceived from a human's lens as three-dimensional. With this, the Magnus effect can deflect the ball slightly to the side, and again impact the final displacement or velocity of the volleyball.
3. I did not have the drag coefficient of my ball and had to use a standard Molten volleyball. Without the equipment to measure the drag coefficient, it limits my exploration to calculate the Magnus effect forces to the values of the true drag coefficient of my volleyball.
4. My results ended with possible systematic errors that hindered these experiments from accurate results. The height of the volleyball net could have been improperly set up, which affects the projectile motion of the volleyball at the instant it goes over the net. Thus, affecting my data collection. The deflation of the volleyball could also be a possibility, and the systematic error is that the entire experiment is flawed as the cross-sectional area of the ball would be changed. A deflated ball would have a decreased area and the decrease in pressure would deform the ball. This increases the effect that air resistance has on the ball as it is a non-streamlined and distorted shape (mtb, 2015).

Future Improvements

For further investigation into this topic, I can analyze the projectile motion of the volleyball serve from a three-dimensional perspective. This would yield better experimental results, as the angular velocity would be more significant in changing the trajectory along the z-axis of a moving ball. Because it is natural for a volleyball to deflate as time passes, I can also improve this experiment by further analyzing different volleyballs with various amounts of pressure or volumes of air. These variables affect the weight and cross-sectional area of the volleyball. Therefore, it allows me to investigate deeper in the topic of the Magnus effect force and how different shapes or magnitudes of volleyballs affect the velocity to go through air resistance to reach a certain point on the court. Collecting additional data points to supplement trials for the exploration would improve the consistency of the final results. Altogether, many improvements can be made in the future if this experiment were to be done again. Nevertheless, the investigation was a success as I was able to answer my aim and show that the x velocity affects the need to spin the ball to reach a fixed target.

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Appendix A

Remaining of the calculations after trial #1.

Calculations With Uncertainties - Trial #2

t (s)	v_x (ms⁻¹)
$= 2.06 \pm \frac{\text{max} - \text{min}}{2}$ $= 2.06 \pm \frac{2.12 - 2.00}{2}$ $= 2.06 \pm 0.06\text{s}$	$= \text{x-displacement} \div t$ $= (18 \pm 1) \div (2.06 \pm 0.06)$ $= 8.7 \pm \left(\frac{1}{18} + \frac{0.06}{2.06}\right) \cdot 8.7$ $= 8.7 \pm 0.7\text{ms}^{-1}$
degrees turned (°)	ω (rs⁻¹)
$= 639 \pm \frac{\text{max} - \text{min}}{2}$ $= 639 \pm \frac{765 - 495}{2}$ $= 639 \pm 135^\circ$	$= \frac{\text{degrees turned} \div 360}{t}$ $= \frac{(639 \pm 135) \div 360}{2.06 \pm 0.06}$ $= 0.86 \pm \left(\frac{135 \div 360}{639 \div 360} + \frac{0.06}{2.06}\right) \cdot 0.86$ $= 0.9 \pm 0.2\text{rs}^{-1}$

Calculations With Uncertainties - Trial #3

t (s)	v_x (ms⁻¹)
$= 1.72 \pm \frac{\text{max} - \text{min}}{2}$ $= 1.72 \pm \frac{1.84 - 1.62}{2}$ $= 1.72 \pm 0.11\text{s}$	$= \text{x-displacement} \div t$ $= (18 \pm 1) \div (1.72 \pm 0.11)$ $= 10 \pm \left(\frac{1}{18} + \frac{0.11}{1.72}\right) \cdot 10$ $= 10 \pm 1\text{ms}^{-1}$
degrees turned (°)	ω (rs⁻¹)
$= 864 \pm \frac{\text{max} - \text{min}}{2}$ $= 864 \pm \frac{990 - 720}{2}$ $= 864 \pm 135^\circ$	$= \frac{\text{degrees turned} \div 360}{t}$ $= \frac{(864 \pm 135) \div 360}{1.72 \pm 0.11}$ $= 1.4 \pm \left(\frac{135 \div 360}{864 \div 360} + \frac{0.11}{2.06}\right) \cdot 1.4$ $= 1.4 \pm 0.3\text{rs}^{-1}$

Calculations With Uncertainties - Trial #4

t (s)	v_x (ms⁻¹)
$= 1.50 \pm \frac{\text{max} - \text{min}}{2}$ $= 1.50 \pm \frac{1.64 - 1.40}{2}$ $= 1.50 \pm 0.12\text{s}$	$= \text{x-displacement} \div t$ $= (18 \pm 1) \div (1.50 \pm 0.12)$ $= 12 \pm \left(\frac{1}{18} + \frac{0.12}{1.50}\right) \cdot 12$ $= 12 \pm 2\text{ms}^{-1}$
degrees turned (°)	ω (rs⁻¹)
$= 1206 \pm \frac{\text{max} - \text{min}}{2}$ $= 1206 \pm \frac{1350 - 1080}{2}$ $= 1206 \pm 135^\circ$	$= \frac{\text{degrees turned} \div 360}{t}$ $= \frac{(1206 \pm 135) \div 360}{1.50 \pm 0.12}$ $= 2.2 \pm \left(\frac{135 \div 360}{1206 \div 360} + \frac{0.12}{1.50}\right) \cdot 2.2$ $= 2.2 \pm 0.4\text{rs}^{-1}$

Calculations With Uncertainties - Trial #5

t (s)	v_x (ms⁻¹)
$= 1.00 \pm \frac{\text{max} - \text{min}}{2}$ $= 1.00 \pm \frac{1.06 - 0.92}{2}$ $= 1.00 \pm 0.07\text{s}$	$= \text{x-displacement} \div t$ $= (18 \pm 1) \div (1.00 \pm 0.07)$ $= 18 \pm \left(\frac{1}{18} + \frac{0.07}{1.00}\right) \cdot 18$ $= 18 \pm 2\text{ms}^{-1}$
degrees turned (°)	ω (rs⁻¹)
$= 1819 \pm \frac{\text{max} - \text{min}}{2}$ $= 1819 \pm \frac{2025 - 1715}{2}$ $= 1819 \pm 155^\circ$	$= \frac{\text{degrees turned} \div 360}{t}$ $= \frac{(1819 \pm 155) \div 360}{1.00 \pm 0.07}$ $= 5.1 \pm \left(\frac{155 \div 360}{1819 \div 360} + \frac{0.07}{1.00}\right) \cdot 5.1$ $= 2.2 \pm 0.8\text{rs}^{-1}$

Appendix B

Magnus Effect Force - Results

Magnus effect force: $F_m = 2\pi S\omega v_x$ where, S = air resistance, ω = angular velocity, v_x = velocity in the x direction.

Air resistance: $S = \frac{1}{2}\rho v_x^2 CA$ where, ρ = density of air, v_x = velocity in the x direction, C = drag coefficient, A = two-dimensional area of the ball.

Therefore, I can continue the process for the following 4 trials.

Trial #2

$$S = \frac{1}{2} \cdot 1.23 \cdot 8.7^2 \cdot 0.17 \cdot 0.035$$

$$S = 0.28\text{N}$$

$$F_m = 2\pi \cdot 0.28 \cdot 0.86 \cdot 8.7$$

$$F_m \approx 13\text{N}$$

Trial #3

$$S = \frac{1}{2} \cdot 1.23 \cdot 10^2 \cdot 0.17 \cdot 0.035$$

$$S = 0.37\text{N}$$

$$F_m = 2\pi \cdot 0.37 \cdot 1.4 \cdot 10$$

$$F_m \approx 35\text{N}$$

Trial #4

$$S = \frac{1}{2} \cdot 1.23 \cdot 12^2 \cdot 0.17 \cdot 0.035$$

$$S = 0.53\text{N}$$

$$F_m = 2\pi \cdot 0.53 \cdot 2.2 \cdot 12$$

$$F_m \approx 88\text{N}$$

Trial #5

$$S = \frac{1}{2} \cdot 1.23 \cdot 18^2 \cdot 0.17 \cdot 0.035$$

$$S = 1.2\text{N}$$

$$F_m = 2\pi \cdot 1.2 \cdot 5.1 \cdot 18$$

$$F_m \approx 690\text{N}$$