Investigating the Q Factor of a Homemade Pendulum

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Abstract

This paper analyzes experiments conducted on a homemade pendulum to validate theoretical predictions. Using a water-filled plastic bottle and a string, several key conclusions were drawn. Firstly, the period was found to depend on the initial angle (except under the small-angle approximation). This dependence is described by the quadratic model where $T = T_0\left(1 + B\theta_0 + C\theta_0^2\right)$ and with experimentally determined parameters: $T_0 = 1.740 \pm 0.003s$, $B = 0.001 \pm 0.001$, and $C = 0.047 \pm 0.002$. The quality factor was determined using two methods yielding: $Q_{\text{by counting}} = 357 \pm 1$ and $Q_{\text{by equation}} = 430 \pm 20$. Then, the period was proven to depend on the pendulum's string length according to the power law function $T = kL^n$, with the scaling parameters: $k_1 = 1.98 \pm 0.01$, $k_2 = 1.99 \pm 0.01$, and $n = 0.497 \pm 0.002$. Despite this, models confirmed that the Q factor and the string length have no correlation.

1 Introduction

The motion of a pendulum can be modelled as a damped harmonic oscillator, which describes how a mass or bob released from rest at an initial angle ($\theta_0 < 90°$), will swing back and forth as expressed by [1]:

$$\theta(t) = \theta_0 e^{-\frac{t}{\tau}} cos(2\pi\frac{t}{T} + \phi_0) \tag{1}$$

In Equation 1, t is time and ϕ_0 is the phase constant, set at the point when the pendulum is released. τ is the time constant, and T is the period, which will later be measured.

To ensure that the pendulum is reliable for future experiments, it is important to test for an asymmetry by arbitrarily choosing one side as positive and releasing the pendulum from both positive and negative initial positions [1].

$$T = T_0\left(1 + B\theta_0 + C\theta_0^2 + ...\right) \tag{2}$$

Whence, B and C should be experimentally zero to verify that the pendulum does not undergo unpredictable motion due to external factors. Based on Equation 1, $\theta_0 e^{-\frac{t}{\tau}}$ describes an exponential decay of the sinusoidal function when plotted. Accordingly, taking the natural logarithm yields a linear plot, where τ can be found:

$$ln(\theta) = -\frac{t}{\tau} + ln(\theta_0) \tag{3}$$

With Equation 3, the quality (Q) factor (ratio of initial energy stored in the pendulum to the energy lost per radian of the oscillation) can thus be calculated [1]:

$$Q = \pi\frac{\tau}{T} \tag{4}$$

Equivalently, the Q factor is dependent on the mass of the bob (m), frequency of the oscillation (ω), and damping force (or opposite frictional forces) per unit velocity (γ) represented by [2]:

$$Q = \frac{m\omega}{\gamma} \tag{5}$$

We can also predict the experimental period compared to a theoretical prediction; let L be the length of the string to the centre of mass of the bob:

$$T = 2\pi\sqrt{\frac{L}{g}} \tag{6}$$

To simplify, the formula can be modified to two variables with the predictions $k = 2$ and $n = 0.5$ [1]:

$$T \approx kL^n \tag{7}$$

This in turn makes it straightforward to verify whether data collected follow the theory by manipulating the equation into a linear regression model using logarithm laws:

$$ln(T) = ln(kL^n) \tag{8}$$

$$ln(T) = nln(L) + ln(k) \tag{9}$$

Ergo, by fitting a graph to mirror Equation 9 where $y = mx + b$, we can compare the experimental k and n with its respective theoretical values.

Finally, to establish whether an experiment verifies theoretical predictions, a relative comparison of an experimental value versus theory, can be expressed using the percentage error formula [3]:

$$\text{Percent Error} = \frac{|\text{Experiment} - \text{Theory}|}{\text{Theory}} \times 100 \tag{10}$$

Overall, the findings make evident that theoretical models reasonably reflect a pendulum's motion in the real world; yet, there is room for improvement in elements that will be further discussed in the paper.

2 Method

The homemade pendulum was constructed by using a plastic bottle posing as the bob. Effectively, water was filled to the maximum capacity shown in Figure 1, to maintain near a constant centre of mass while in motion; this prevents varying mass distribution from the possibility of water sloshing and thus controls random uncertainty.

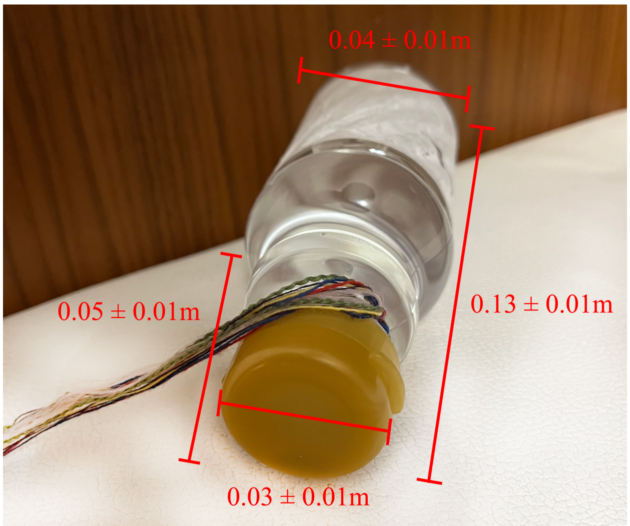

Figure 1. Water bottle used to act as the bob of the pendulum. Dimensions are approximated as two cylinders stacked on top of each other. Marked lengths are not to scale.

Then, a string with negligible mass was tied to the cap of a second water bottle of similar mass for stability. The length of the string to the centre of mass (by visual estimation) of the swinging bottle was $0.75 \pm 0.01m$ (adding the $0.005m$ uncertainty of both sides of a metre stick). To avoid incurring additional costs, the top of the string was attached with tape to a pull-up bar to act as the pivot of the pendulum swinging from left to right, as seen in Figure 2.

With the *Python* Uncertainties Package, a customized Python script was used to generate the plots from the processed data points visually, calculate the uncertainties, and display error bars [4].

2.1 Period vs. Angle

The pendulum must first be tested in case of asymmetry, by comparing how period changes from negative to positive angles, to justify its reliability as a model. To investigate the dependence of period on the angle, five trials were recorded: from -80° to -10° and 10° to 80° , in increments of 10°.

Methodically, the bob was released at each target angle and videotaped to track the time for 10 oscillations—dividing by 10 gave the period. No Type A uncertainties were associated with each set angle; a protractor's uncertainty of $\Delta\theta = 0.009rad$ was thus used instead. The protractor was securely taped to confirm the release angle relative to the pivot point.

Simultaneously, a stopwatch was used to measure the time to complete 10 oscillations. This gives a reaction time uncertainty of $\Delta t = 0.2s$, in which the total time was divided by 10 oscillations. After averaging five trials from each angle, Type A period uncertainties were taken; they were greater than the bias reaction time.

2.2 Q Factor

Using the Open Source Physics *Tracker*, a few thousand frames each from five videos were gathered; Q factor will be measured after plotting the data points retrieved [5]. After a few preliminary tests, it was noticeably important to wrap the bottle in a bright green colour illustrated in Figure 2, to precisely detect the bob's position, which reduces random error of the *Tracker*.

Figure 2. Two water bottles are tied to a pull-up bar to provide a counterweight system, allowing length adjustments of the string from the pivot point to the centre of mass, when need be.

The five trials are in 30fps, where the time uncertainty is half a frame: $\Delta t = 0.02s$. The *Tracker* may also track both sides of the cap sporadically. Then, knowing that the cap radius is $\approx 0.015m$ as seen in Figure 1, we can estimate the angle uncertainty—assuming the cap is an arc length—which equals the radius (length of string) times angle: $\Delta\theta = \frac{0.015}{0.75} = 0.02rad$.

Each video shows the bob released at an initial angle near 90° to examine its predicted exponential decay. To measure the Q factor, it is important to begin the calculations at an angle below the small-angle approximation, which will be further discussed. Then, two methods will be used: counting the amplitudes until the pendulum decays to $e^{-\frac{\pi}{2}}$ or $\approx 21\%$ of its initial value, and using Equation 4.

2.3 Period vs. Length

Five trials of string lengths from $0.15 \pm 0.01m$ to $1.00 \pm 0.01m$, in increments of $0.05m$ were conducted. This was done by releasing the pendulum at each length, then following the same procedure in Section 2.1 to find the corresponding period. Displayed in Figure 2.2, the two-bottle counterweight system enables adjustments in the string length as desired.

Subsequent experiments will be run using one constant angle of $0.150 \pm 0.009rad$ (uncertainty based on the protractor), less than the small-angle approximation found in Section 3.1.1, ensuring that the C value and imperfections of the pendulum do not influence our findings.

There were no Type A uncertainties associated with each string length; rather, a metre stick's uncertainty $\pm 0.01m$ was used. Simultaneously, a stopwatch measured the time taken to complete 10 oscillations for each trial. This gives a reaction time uncertainty of $\Delta t = 0.2s$, same as Section 2.1. After averaging the five trials for each length, Type A uncertainties in the period were taken, as most exceeded than the bias time uncertainty.

2.4 Q Factor vs. Length

Q factor for the string length of $0.75 \pm 0.01m$ will be remeasured alongside to maintain consistency with the other lengths. Accordingly, the same number of lengths was used as in the previous experiment and each recorded for $100s$.

After observing the Q factor results in Section 3.4.1, although the counting method presents a lower uncertainty, presumably, the Q factor derived from Equation 3 is more reliable, because the trend generally follows all points. In contrast, the counting method provides merely a few points within a limited range close to the 21% decay, which could be outliers or inconsistent with the general trend; the damping may also be uneven at the specific points. Hence, during length and Q factor measurements, the equation method was used for efficiency.

As justified in Section 2.3, we must release all initial angles consistently at $0.15 \pm 0.02rad$—uncertainty is based on the *Tracker*, calculated in Section 2.2. The method of conducting the experiment will follow Section 2.2, using the *Tracker*.

3 Data Analysis

3.1 Period vs. Angle

As illustrated in Figure 3, the plot is fitted to a quadratic function. Before continuing this investigation, we must find the coefficients required to test in case of asymmetry using Equation 2.

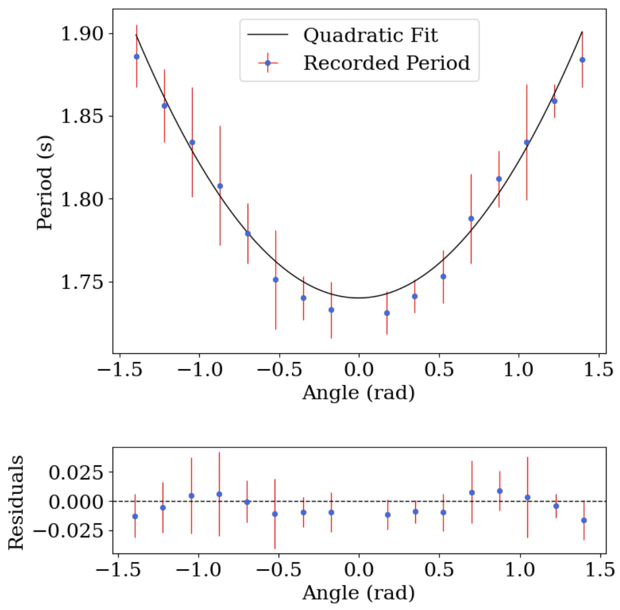

Figure 3. Above: a plot of the period as a function of the angle when fitted to a quadratic function with $R^2 = 0.98$. To test for symmetry, angles increase from negative to positive from left to right. Below: a representation of the residuals of the fit. Note, angle uncertainties are too small to be seen.

Referring to the graph above, by factoring T_0 from Equation 2 and using *Python*, the coefficients needed for analysis were obtained from the line of best fit:

$$T_0 = 1.740 \pm 0.003s \tag{11}$$

$$B = 0.001 \pm 0.001 \tag{12}$$

$$C = 0.047 \pm 0.002 \tag{13}$$

3.1.1 Discussion

B is experimentally zero as its uncertainty is twice the non-rounded value ($B = 0.0005$). In later sections, angles used must be small enough so C can also be ignored. Moreover, the $R^2 = 0.98$ in Figure 3 mirrors a strong regression model with minimal variation.

Altogether, releasing the bob at the same absolute angle from both sides thus presents negligible differences between the period. The homemade pendulum is thus symmetrical, with merely statistical uncertainties causing the fluctuations.

Next, in Figure 3, we can take the small-angle approximation as $|\theta| < 0.5rad$; a straight horizontal line can pass through all six error bars simultaneously, where $sin\theta \approx \theta$ and the period stays constant.

Noticeably, at larger angles, the period is dependent on the corresponding angle of release. This is true in a quadratic regression model, yet less than the small-angle approximation, period is independent.

Generally, the data collection could be improved by increasing the number of trials. Increasing the camera and the *Tracker* detection frame rate could also be beneficial in minimizing the data points' variation.

3.2 Q Factor

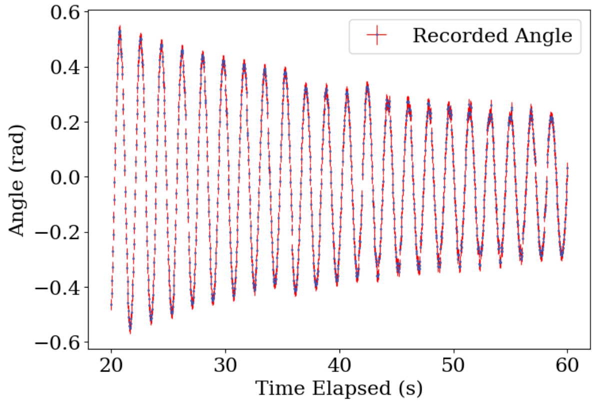

Figure 4. A plot of the angle as a function of the time elapsed. Selected data points that qualitatively resemble an exponential decay of the pendulum in the first 20 to 60 seconds. A line of best fit is not needed for this illustration. Residuals are thus not present.

Examining many snippets of time elapsed versus angle, such as Figure 4, we can confirm that an exponential decay occurs.

Accordingly, we can create a new graph in Figure 5 with an initial angle of $0.15 \pm 0.02rad$, to adhere to values less than the small-angle approximation. This was done to avoid unintended energy loss to surroundings from larger angles, thus creating a new model as close to a linear function as possible to estimate τ, then calculate the Q factor—uncertainties were propagated by *Python* providing a graph with error bars as well.

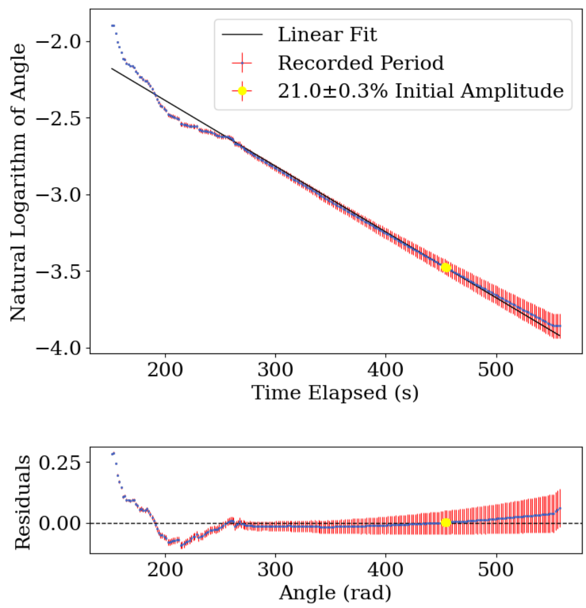

Figure 5. Above: a plot of the natural log of angle as a function of the time elapsed when fitted to a linear function with $R^2 = 0.99$. Yellow dots represent the initial amplitude decayed to $\approx 21.0 \pm 0.3\%$. Negative reciprocal of slope gives τ. Below: a representation of the residuals of the fit. Note, time uncertainties are too small to be seen.

By counting the amplitudes—for instance the peaks in Figure 4—until it decayed to $e^{-\frac{\pi}{2}}$ or $\approx 21.0 \pm 0.3\%$ of its initial value, we attain two data points closest to this decay percentage (yielding a 0.3% uncertainty between both values). In turn, we can take the average value where $\frac{Q}{2} = 178.5$:

$$Q_{\text{by counting}} = 357 \pm 1 \tag{14}$$

Using the plot, we get the negative reciprocal of the slope: time constant (τ), and the inherent period (T) is the average period of small angles:

$$\tau = 234 \pm 9 \tag{15}$$

$$T = 1.71 \pm 0.03s \tag{16}$$

Using Equation (4), τ and T, we can isolate for Q:

$$Q_{\text{by equation}} = 430 \pm 20 \tag{17}$$

With both Q factors, we can apply uncertainty propagation rules to assess their difference error [6].

$$\Delta Q = 70 \pm 20 \tag{18}$$

3.2.1 Discussion

The difference error in Equation 18 signifies that the natural logarithm of angle and time elapsed does not exhibit a perfectly linear trend, or a representation of a decaying exponential function as predicted. Next step: a more advanced software could be used instead of *Tracker*, to minimize faulty detections and random error.

Besides, the difference suggests that energy loss is still present despite how the pendulum was tracked within these small angles. In the future, the initial angle could be even smaller to ensure that the value of C in Figure 3 can be ignored; the energy loss to the surroundings from resistance to air or other external factors that could have distorted the predicted damped motion at larger angles are negligible at even smaller angles.

Then, the $R^2 = 0.99$ in Figure 5 mirrors the conclusion of Section 3.1.1 where most data points precisely fit the regression model, producing a τ without significant error and preventing skew in the calculations of both Q factors.

Overall, the errors and uncertainties found may be attributed to displacements of the string pivoting, friction, or air resistance when swinging. Accordingly, a more aerodynamic bob mass could be substituted, and the string can be set in place more securely rather than only using tape. With a more flexible budget, an alternative to a pull-up bar would be more effective to minimize this friction energy loss.

3.3 Period vs. Length

Fundamentally, to establish whether the period and length have any correlation, the raw data for each point are plotted in Figure 6.

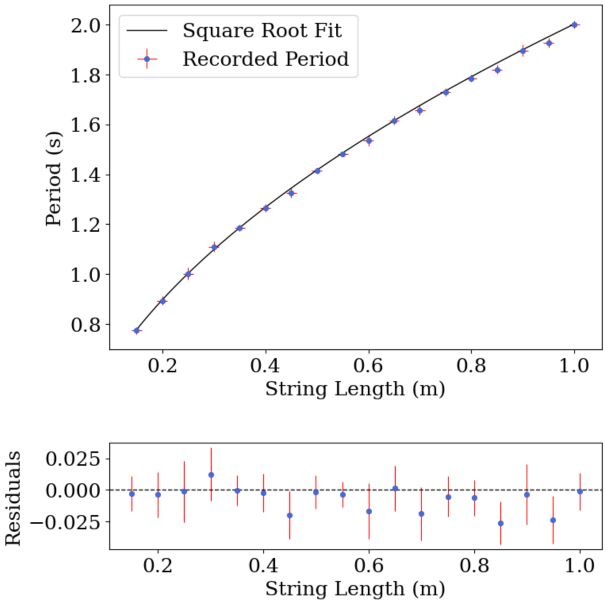

Figure 6. Above: a plot of the period as a function of the string length when fitted to a square root function with $R^2 = 0.99$. Below: a representation of the residuals of the fit. Note, uncertainties of both variables are present but may not be visible.

This square root regression model can then be manipulated by taking the square root of the string length shown in Equation 7, and calculating the slope (k_1) of Figure 7.

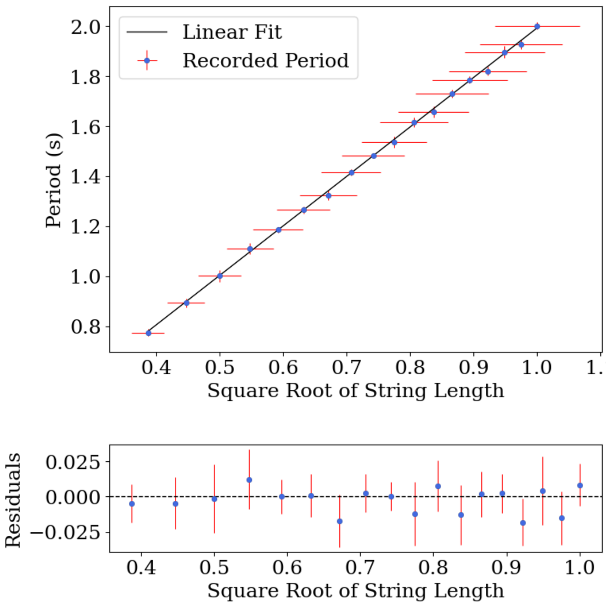

Figure 7. Above: a plot of the period as a function of the square root of the string length when fitted to a linear function with $R^2 = 0.99$. Below: a representation of the residuals of the fit. Note, period uncertainties are too small to be seen.

With Figure 7, we can thus compare the $k_{\text{theoretical}} = 2$ with $k_1 = $ slope:

$$k_1 = 1.98 \pm 0.01 \tag{19}$$

Furthermore, when the natural logarithm of both axes is plotted in Figure 8 we can take $e^{\text{y-intercept}}$ to confirm another k_2 as well, alongside an n to compare with $n_{\text{theoretical}} = 0.5$; all refer to Equation 9.

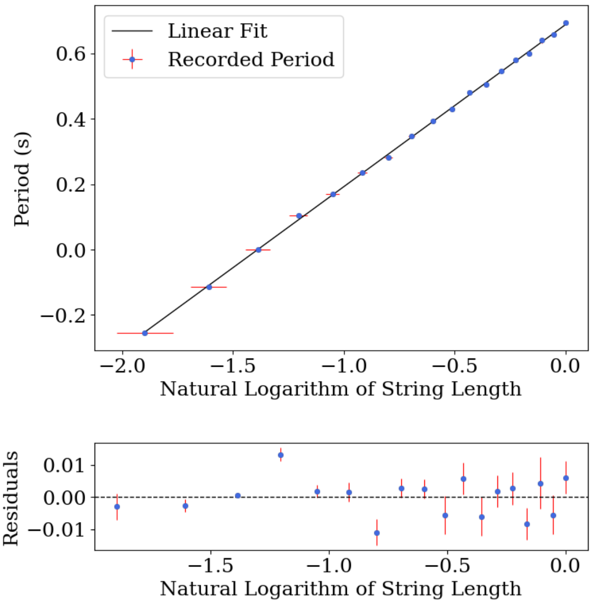

Figure 8. Above: a plot of the natural logarithm of period as a function of the natural logarithm of string length when fitted to a linear function with $R^2 = 0.99$. Below: a representation of the residuals of the fit. Note, uncertainties of both variables are present but may not be visible.

From Figure 8, the experimental values as aforementioned are generated using *Python*:

$$n = 0.497 \pm 0.002 \tag{20}$$

$$k_2 = e^{\text{y-intercept}} \tag{21}$$

$$k_2 = 1.99 \pm 0.01 \tag{22}$$

Subsequently, we can compare theoretical and experimental values to validate Equation 7 predicting $k = 2$ and $n = 0.5$, to identify the accuracy of the lab. This relative comparison can be expressed using the percentage error formula from Equation 10 (below, k_1 was displayed instead of k_2 as it has the higher percentage error):

$$\text{Percentage Error of } k = 1\% \tag{23}$$

$$\text{Percentage Error of } n = 0.6\% \tag{24}$$

3.3.1 Discussion

Therefore, the period is dependent on the string length, and we can see that the data points fit the theoretical prediction and Equation 7 to a high degree.

Comparing the theoretical prediction of k and n, it is evident that a percentage error of 0.1% and 0.6% respectively are considered sufficiently small to be neglected. With this, we can say that period versus string length of the homemade pendulum accurately reflects the theoretical model and correlation expressed as Equation 7. The $R^2 = 0.99$ also reinforces the relationship proven between the two variables, especially since the variance in the data points is close to none.

To extend this experiment, if there is room below the pendulum, if we increase the range of string length magnitudes we can confirm to a higher degree if the relationship holds even at greater lengths where energy loss is much more prevalent. Similarly, a greater number of trials is necessary to reduce random error, providing an even more reliable estimation of k and n in the future.

3.4 Q Factor vs. Length

As reinforced in Section 2.4, the equation method was used for all Q factor versus string length trials.

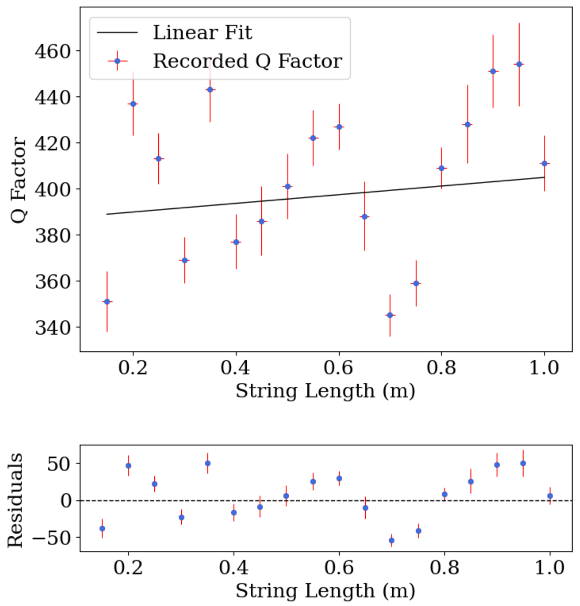

Figure 9. Above: a plot of the Q factor as a function of the string length when fitted to a linear function with $R^2 = 0.1$. Below: a representation of the residuals of the fit. Note, string length uncertainties are too small to be seen.

Visually in Figure 9, and the $R^2 = 0.1$ suggests that there is no correlation. To verify this, we can use *Python* to generate the uncertainty of the slope (m):

$$m = 20 \pm 30 \tag{25}$$

3.4.1 Discussion

As expressed, the uncertainty of the slope is nearly twice the slope itself, illustrating no statistically significant correlation. The $R^2 = 0.1$ also tells us the variance is sporadic; ergo, it is inconclusive that Q factor depends on the string length of the pendulum.

Conversely, energy loss is attributed to friction at the pivot and air resistance where longer string lengths cause the pendulum's period to increase, as shown in Figure 6; this change in period should in fact decrease the frequency of oscillations.

Accordingly, the string length should optimally influence the Q factor, largely contradicting our model. This is because as specified by the Q factor in Equation 5, Q factor is rather directly proportional to the frequency of the oscillation, which relates to period and length as verified online [7].

In future iterations, a larger dataset could be beneficial, as discussed in Section 3.3.1. Although fixing the theoretical contradiction of the homemade model may be difficult, we can consider minimizing energy loss using methods discussed in Section 3.2.1 to yield closer to the expected predictions in the future.

4 Conclusion

Ultimately, the period depends on the initial angle. However, period is independent when using the small-angle approximation. Additionally, the homemade pendulum was quantitatively proven to be symmetrical ($B \approx 0$ within uncertainty), confirming the theory that the bob will swing back and forth evenly.

Correspondingly, two Q factors found by counting and using an equation were validated at the smaller angles, and assuming constant period. However, the exponential decay plot did not fit the linear regression model perfectly; factors such as irregular friction or faulty detections by the *Tracker* should be minimized in the future.

Next, we proved that the period depends on the string length, where quantitatively the experimental model fit almost perfectly with the theoretically predicted values: $k = 2$ and $n = 0.5$.

Despite this trend, experimentally, Q factor values have no correlation with the pendulum's string length. Our model proved to contradict theoretical predictions where they should be directly proportional to each other.

For future experiments, an even smaller angle for all Q factor determinations could be taken to reduce external factors. In essence, while the homemade pendulum revealed conclusions parallel to its theory and already established equations, improving the data through aspects such as trial repetition and better technology could further enhance the reliability of the results.

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Appendix A: Python Usage

OpenAI was used to fine-tune the Python code provided by the *Python* uncertainties package, in which the script was run through the open-source computing environment, Pyzo.